Adaptive Memoryless Observer Design for Nonlinear Time-Delay Systems

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Abstract— **Two memoryless adaptive observers are proposed for systems with unknown time-delays and nonlinearities, in which time-delays are not used in the observer. Using high order neural networks (HONN), the precise system model, the Lipschitz, linear-in-parameter or norm-bounded assumptions of nonlinear functions are not needed. A robust term based on the matching condition is introduced in the first observer, such that the observer error converges to the origin. The observer error of the second scheme without the robust compensation is proved to be semi-globally uniformly ultimately bounded (SGUUB). Simulations verify the effectiveness of the proposed methods.**

I. Introduction

Time-delays as well as nonlinearities are often encountered in various systems, which render the control design more difficult [1]. During the past decades, a lot of significant advances have been proposed in stability analysis and feedback control for time-delay systems, e.g., [1]-[4] and references therein. Among these schemes, the system states are assumed to be precisely known for the control design, which is not true in some practical cases as some commercial control systems are not equipped with enough sensors. This inspires the issue of observer design for control systems, which is an active research topic in the control community.

Different types of observers have been proposed, e.g., Luenberger observer [5]-[6], adaptive observer [7]-[8], high-gain observer [9]-[10] and among others. Robust observers [11]-[15] and sliding mode observers [16]-[20] were developed to deal with parameter uncertainties and unknown nonlinearities. Most of them, as pointed out in [7], exploit the existence conditions of observers by means of linear matrix inequalities (LMI) or matrix inequalities (MI). Moreover, the Lipschitz or linear-in-parameter condition of nonlinear functions or the upper bounds of uncertainties are utilized. To relax the requirement of nonlinear functions, neural networks (NN) have been used for observer design since the past decade. Kim & Lewis developed neural-based observers for Brunovsky canonical systems in [21]-[22]. The employed strictly positive real (SPR) condition was further studied in [23] by using a novel observer gain design method. An adaptive neural-based error observer for second-order systems was also proposed in [24], which is combined with the control design. Another NN observer[25] was constructed for systems in which nonlinearities depend only upon the output measurement. The recent work of Abdollahi *et al.* [26] investigated neural observers for more general nonlinear systems. However, it is noted that the effect of time-delay is not considered in the aforementioned NN-based observers.

For time-delay systems, Lyapunov-Krasovskii functions were introduced in the observer design, e.g. [7], [11]-[15], [19], to prove the stability of the observer error system. As can be seen, the delayed dynamics injected into the system are all assumed to be linear term with constant matrices [11], [15], [17] or bounded uncertainties [12]-[14], [19]. In addition, time-delays are usually involved in the observer realization [11]-[12], [19], which means the value of delays should be known precisely. To the best of our knowledge, little attention has been paid to the memoryless observer design for nonlinear systems with time-delays [13]-[15], which is independent of the delays. Specifically, if the delayed states are involved in the unknown system nonlinearities, the memoryless observer design for such systems has not been fully investigated.

In this paper, we study the adaptive memoryless observer design for nonlinear systems with unknown time-varying delays. Novel appropriate Lyapunov-Krasovskii functions are introduced to compensate for the effect of time-delays and high-order neural networks (HONN) are employed to deal with unknown dynamics. If the matching condition (strictly positive real condition) holds, an adaptive robust term can be proposed to compensate for the effect of NN approximation error, such that the observer error is asymptotically stable. An alternative observer without using the matching condition is subsequently proposed, where the observer error is proved to be semi-globally uniformly ultimately bounded (SGUUB). The precise system model, the value of delays or other restrictive assumptions (i.e. norm-bounded[9]-[16], Lipschitz [5]-[6] and linear-in- parameters [14], [19], etc.) of unknown nonlinear functions are not needed. The existence conditions of the proposed observers are obtained explicitly, which provides the possibility for further analysis on the dynamical observation performance. Comparative simulations are included to illustrate the effectiveness of the schemes.

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II. PROBLEM STATEMENT

Consider nonlinear time-delay systems described by

$$\begin{cases} \dot{x}(t) = Ax(t) + B[f(x(t),u(t)) + h(x(t-\tau(t)))] + Du(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x = [x_1, x_2 \cdots x_n]^T \in \mathbb{R}^n$, $u \in \mathbb{R}^q$ and $y \in \mathbb{R}^p$ are the bounded system states, control input and output, respectively. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{n\times q}$ are system matrices. Functions $f(x(t),u(t)) \in \mathbb{R}^m$, $h(x(t-\tau(t))) = [h_1(x(t-\tau_1(t))), \cdots, \quad h_m(x(t-\tau_m(t)))]^T \in \mathbb{R}^m$ are unknown but continuous. The unknown time-delays $\tau(t) = [\tau_1(t), \cdots, \tau_m(t)]$ are bounded by $\tau_i(t) \le \tau_M$ and $\dot{\tau}_i(t) \le \bar{\tau} < 1$, where τ_M and $\bar{\tau}$ are the upper bounds of delays and their varying rates.

High-order neural networks (HONN) can approximate nonlinear functions up to arbitrary accuracy on a compact set Ω_Z, which is specialized as [29]

$$F(Z) = W^{*T}\Phi(Z) + \varepsilon, \quad \forall Z \in \Omega_Z \subset \mathbb{R}^n \tag{2}$$

with ideal weights $W^* \in \mathbb{R}^l$ and approximation error $\varepsilon \in \mathbb{R}$, which are bounded by $\|W^*\| \le W_N$ and $|\varepsilon| \le \varepsilon_N$. $\Phi(Z) \in \mathbb{R}^l$ is a vector with element $\Phi_k(Z) = \prod_{j\in J_k}[\sigma(Z_j)]^{d_k(j)}, k = 1,\ldots,l$, where J_k are collections of l not ordered subsets of $\{0,1,\ldots,n\}$ and $d_k(j)$ are nonnegative integers. The activation function $\sigma(x)$ is a sigmoid function.

The following lemmas are useful for stability analysis [3]:

Lemma 1: For any constant $\omega > 0$ and any variable $z \in \mathbb{R}$, we have $\lim_{z\to 0} \tanh^2(z/\omega)/z = 0$.

Lemma 2: Define a set as $\Omega_e = \{z \mid |z| < 0.8814\omega\}$, then for any $z \notin \Omega_e$, the inequality $1 - 2\tanh^2(z/\omega) \le 0$ holds.

III. OBSERVER DESIGN USING ROTUST TERM

In this section, an adaptive observer with a robust compensation term is first proposed, where asymptotic stability of the observer error is achieved. For simplicity, we will omit the time variable t, except with time-delay $\tau(t)$.

The following assumption is used in this section:

Assumption 1: The system matrices B and C satisfy $PB = C^T F^T \in \mathbb{R}^{n\times m}$ for a matrix $F \in \mathbb{R}^{m\times p}$ and a positive definite matrix P, where (P,Q) is a Lyapunov matrix pair.

Remark 1: Assumption 1 is usually considered as the strictly positive real (SPR) type assumption [16], [21]-[22], [31]. Walcott *et al*. [16] and Herrmann *et al*. [31] provide a detailed analysis for the construction of matrices F and Q to fulfill this condition.

The following uniformly continuous and bounded positive functions $\sigma_i(t) > 0$ are used

$$\lim_{t\to\infty}\int_0^t \sigma_i(\varsigma)d\varsigma \le \bar{\sigma}, \qquad i = 1,2,3 \tag{3}$$

where $\bar{\sigma} > 0$ is a positive bounded constant.

For system (1), the following observer can be proposed

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B\left[\hat{W}^T\Phi(\hat{x}(t),u(t),\hat{y}(t)) + S(\tilde{y}(t),t)\right] \\ \qquad +Du(t) + K\left[y(t) - \hat{y}(t)\right] \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{4}$$

where $K \in \mathbb{R}^{n\times p}$ is the gain matrix, such that $A_0 = A - KC$ is stable, i.e. we have $A_0^T P + PA_0 = -Q$ for symmetric positive definite matrix P and Q. $\hat{W} = [\hat{W}_1, \cdots, \hat{W}_n] \in \mathbb{R}^{l\times m}$ is the NN weight matrix with $\hat{W}_j = [\hat{w}_{j1}, \cdots, \hat{w}_{jl}]^T \in \mathbb{R}^l$, $j = 1,\cdots,m$ and $\Phi(\hat{x},u) = [\Phi_1, \cdots, \Phi_l]^T \in \mathbb{R}^l$ is the basis function vector.

The adaptive robust compensation term is

$$S(\tilde{y}(t),t) = \frac{F\tilde{y}(t)}{2\delta(t)} + \frac{\hat{\varepsilon}^2(t)F\tilde{y}(t)}{\hat{\varepsilon}(t)\|F\tilde{y}(t)\| + \sigma_2(t)} \tag{5}$$

where $\tilde{y}(t) = y(t) - \hat{y}(t)$ is the output error, $\sigma_2(t)$ is a positive function given in (3) and $F \in \mathbb{R}^{m\times p}$ is a design matrix fulfilling Assumption 1. $\delta(t)$ is a positive function bounded by $\delta(t) \le \bar{\delta}(t) = \min\{\sigma_2(t-\tau_1), \sigma_2(t-\tau_2), \cdots, \sigma_2(t-\tau_m)\}$. Provided $\sigma_2(t)$ is chosen to be a decreasing function, we can select $\delta(t) = \sigma_2(t)$ to fulfill this condition.

The adaptive parameter $\hat{\varepsilon}(t)$ and NN weights $\hat{W}$ in (5) are given by

$$\dot{\hat{W}} = \Gamma\Phi(\hat{x},u)\tilde{y}^T F^T - \sigma_1(t)\Gamma\hat{W} \tag{6}$$

$$\dot{\hat{\varepsilon}} = \Upsilon\|F\tilde{y}(t)\| - \sigma_3(t)\Upsilon\hat{\varepsilon}, \tag{7}$$

with $\Gamma = \Gamma^T > 0$, $\Upsilon > 0$ the learning parameter and $\sigma_i(t)$ is the positive functions in (3) as σ - modification parameter.

Define the observer error as $\tilde{x}(t) = x(t) - \hat{x}(t)$, then we can obtain the error equation as

$$\begin{cases} \dot{\tilde{x}}(t) = A_0\tilde{x}(t) + B\left[f(x(t),u(t)) + h(x(t-\tau))\right. \\ \qquad \left. -\hat{W}^T\Phi(\hat{x}(t),u(t),\hat{y}(t)) - S(\tilde{y}(t),t)\right] \\ \tilde{y}(t) = C\tilde{x}(t) \end{cases} \tag{8}$$

Then we have the following result:

Theorem 1: If the matching condition $PB = C^T F^T$ holds for nonlinear time-delay system (1) with the proposed observer (4) ~ (6), then the observer error in (8) converges to zero and all signals in the observer are bounded.

Proof: Select a Lyapunov function as

$$\begin{aligned} V = &\frac{1}{2}\tilde{x}^T P\tilde{x} + \frac{1}{2(1-\bar{\tau})}\sum_{i=1}^m \int_{t-\tau_i}^t \sigma_2(\varsigma)h_i^2(x(\varsigma))d\varsigma \\ &+ \frac{1}{2}tr(\tilde{W}^T\Gamma^{-1}\tilde{W}) + \frac{1}{2}\Upsilon^{-1}\tilde{\varepsilon}^2 \end{aligned} \tag{9}$$

where $\tilde{W} = W^* - \hat{W}$ is the NN weight error and $\tilde{\varepsilon} = \varepsilon_N - \hat{\varepsilon}$

is the NN approximation error with ε_N a positive constant.

Consider the fact $\tau_i \le \tau_M$ and $\dot{\tau}_i \le \bar{\tau} < 1$, we can get the derivative of the Lyapunov-Krasovskii function as

$$\begin{aligned}\dot{V}_2 &= \frac{1}{2(1-\bar{\tau})}\sum_{i=1}^{m}\left(\sigma_2(t)h_i^2(x(t)) - (1-\dot{\tau}_i)\sigma_2(t-\tau_i)h_i^2(x(t-\tau_i))\right) \\ &\le \frac{1}{2(1-\bar{\tau})}\sigma_2(t)h^T(x)h(x) - \frac{1}{2}\bar{\delta}(t)h^T(x(t-\tau))h(x(t-\tau))\end{aligned} \quad (10)$$

By using the matching condition $PB = C^T F^T$, we know that $\tilde{x}^T PB = \tilde{y}^T F^T$. Moreover, from Young's inequality $\pm a^T b \le (a^T a + k^2 b^T b)/2k$ with $k > 0$ and $\delta(t) \le \bar{\delta}(t)$, the derivative of $\dot{V}$ along (6) ~ (8) can be given as

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T PB[f(x,u) - \hat{W}^T\Phi(\hat{x},u,\tilde{y}) - S(\tilde{y},t)] \\ &\quad + \frac{\|F\tilde{y}\|^2}{2\bar{\delta}} + \frac{\sigma_2}{2(1-\bar{\tau})}h^T(x)h(x) + tr(\tilde{W}^T\dot{\tilde{W}}) + \tilde{\varepsilon}\dot{\tilde{\varepsilon}} \\ &\le -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T PB[Q(Z) - \hat{W}^T\Phi(\hat{x},u,\tilde{y})] - \frac{\hat{\varepsilon}^2\|F\tilde{y}\|^2}{\hat{\varepsilon}\|F\tilde{y}\| + \sigma_2} \\ &\quad - tr(\tilde{W}^T\Phi(\hat{x},u,\tilde{y})\tilde{y}^T F^T) + \sigma_1 tr(\tilde{W}^T\hat{W}) - \tilde{\varepsilon}\|F\tilde{y}\| + \sigma_3\tilde{\varepsilon}\hat{\varepsilon} \\ &\quad + \frac{\sigma_2}{2(1-\bar{\tau})}\left[1 - 2\tanh^2\left(\frac{\|F\tilde{y}\|^2}{\omega}\right)\right]h^T(x)h(x)\end{aligned} \quad (11)$$

where $Q(Z) = f(x,u) + \frac{\sigma_2 F\tilde{y}}{(1-\bar{\tau})\|F\tilde{y}\|^2}\tanh^2\left(\frac{\|F\tilde{y}\|^2}{\omega}\right)h^T(x)h(x)$ is an unknown function vector with $\omega > 0$. According to Lemma 1, $Q(Z)$ is well-defined including the point $\|F\tilde{y}(t)\| = 0$, and thus can be approximated by a HONN as

$$Q(Z) = \hat{W}^T\Phi(\hat{x},u,\tilde{y}) + \tilde{W}^T\Phi(\hat{x},u,\tilde{y}) + \tilde{\varepsilon} \quad (12)$$

where $\tilde{\varepsilon} = W^{*T}(\Phi(x,u,\tilde{y}) - \Phi(\hat{x},u,\tilde{y})) + \varepsilon$ is the NN error. Similar to [27] and [28], we can prove that: 1) $\Phi^T\Phi \le l$, where l is the number of neurons; 2) $\tilde{\Phi}^T\tilde{\Phi} \le 4l$, where $\tilde{\Phi} = \Phi(x,u,\tilde{y}) - \Phi(\hat{x},u,\tilde{y})$. Consequently, $\tilde{\varepsilon}$ is bounded by $\|\tilde{\varepsilon}\| \le \varepsilon_N$ with ε_N a positive constant.

Consider the fact $tr(AB^T) = B^T A$ for $A, B \in \mathbb{R}^n$, it follows

$$\begin{aligned}&tr(\tilde{W}^T\Phi\tilde{y}^T F^T) = \tilde{y}^T F^T\tilde{W}^T\Phi = \tilde{x}^T PB\tilde{W}^T\Phi, \\ &tr(\tilde{W}^T\hat{W}) \le W_N\|\tilde{W}\| - \|\tilde{W}\|^2 \le -\|\tilde{W}\|^2/2 + W_N^2/2, \\ &\tilde{\varepsilon}\hat{\varepsilon} \le \varepsilon_N\|\tilde{\varepsilon}\| - \|\tilde{\varepsilon}\|^2 \le -\|\tilde{\varepsilon}\|^2/2 + \varepsilon_N^2/2.\end{aligned} \quad (13)$$

Moreover, the function $h^T(x)h(x)$ is bounded as system states $x(t)$ are bounded in a compact set and $h(x)$ is uniformly continuous. We also know that the relation $\left|1 - 2\tanh^2\left(\|F\tilde{y}(t)\|^2/\omega\right)\right| \le 1$ holds for any $\|F\tilde{y}(t)\| \in \mathbb{R}$.

Then the last term in (11) is bounded by

$$\left\|\left[1 - 2\tanh^2\left(\frac{\|F\tilde{y}(t)\|^2}{\omega}\right)\right]h^T(x)h(x)\right| \le M \quad (14)$$

where M represents a positive constant.

In addition, the following inequality also holds

$$0 \le \frac{ab}{a+b} \le a, \qquad \forall a,b > 0 \quad (15)$$

From (12) ~ (15), we can rewrite (11) as

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \hat{\varepsilon}\|F\tilde{y}\| - \frac{\hat{\varepsilon}^2\|F\tilde{y}\|^2}{\hat{\varepsilon}\|F\tilde{y}\| + \sigma_2} + \frac{1}{2}\left[\sigma_1 W_N^2 + \sigma_3\varepsilon_N^2 + \frac{\sigma_2 M}{1-\bar{\tau}}\right] \\ &\le -\frac{1}{2}\lambda_m(Q)\|\tilde{x}\|^2 + \sigma_2 + \frac{1}{2}\left[\sigma_1 W_N^2 + \sigma_3\varepsilon_N^2 + \frac{\sigma_2 M}{1-\bar{\tau}}\right] \\ &\le -\eta_1\|\tilde{x}\|^2 + \beta_1\sigma(t)\end{aligned} \quad (16)$$

with $\eta_1 = \lambda_m(Q)$ is the minimum eigenvalue of matrix Q, and $\sigma(t) = \max\{\sigma_1(t), \sigma_2(t), \sigma_3(t)\}$, $\beta_1 = 1 + \frac{1}{2}W_N^2 + \frac{1}{2}\varepsilon_N^2 + \frac{M}{2(1-\bar{\tau})}$ are all positive. From (3), we know $\sigma(t)$ is also bounded. Then according to Lyapunov's Theorem, we can obtain $V(t)$, $\tilde{x}(t)$, $\tilde{W}(t)$, $\tilde{\varepsilon}(t)$ and $\tilde{y}(t)$ are bounded, which means $\hat{x}(t)$, $\hat{W}(t)$, $\hat{y}(t)$ and $\hat{\varepsilon}(t)$ are also bounded. Consequently, $\dot{\tilde{x}}(t)$ is bounded according to (8), i.e. $\dot{\tilde{x}}(t) \in L_\infty$.

Moreover, integrating both sides of (16), it can be deduced

$$\begin{aligned}\lim_{t\to\infty}\int_0^t \|\tilde{x}(\varsigma)\|^2 d\varsigma &\le \frac{1}{\eta_1}V(0) + \frac{1}{\eta_1}\lim_{t\to\infty}\int_0^t \sigma(\varsigma)d\varsigma \\ &\le \frac{V(0)}{\eta_1} + \frac{\beta_1\bar{\sigma}}{\eta_1}\end{aligned} \quad (17)$$

It is shown in (17) that, $\tilde{x}(t)$ is square integrable for any bounded initial condition $V(0)$, i.e. $\tilde{x}(t) \in L_2$. Recalling Barbalat's Lemma, we can conclude that $\lim_{t\to\infty}\tilde{x}(t) = 0$. □

Remark 2: Different to previous proposed observers for time-delay systems [11] - [12], [19], the current observation $\hat{x}(t)$, $\tilde{y}(t)$ are employed in the observer (4), and thus the time-delay $\tau(t)$ is not needed in the observer. Moreover, the robust term $S(\tilde{y}(t), t)$ in (4) is utilized to deal with the NN error, such that the asymptotic stability is achieved.

IV. Observer Design without Robust Term

Without the matching condition $PB = C^T F^T$, a generic memoryless observer design is provided as

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{W}^T\Phi(\hat{x}(t), u(t)) + Du(t) + K(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t)\end{cases} \quad (18)$$

where $K \in R^{n\times p}$, $\hat{W} \in R^{l\times m}$ are defined as ones in (4).

The adaptive updating-law of NN in (18) is given as

$$\dot{\hat{W}} = \Gamma\Phi(\hat{x},u)\tilde{y}^T F^T - \sigma\Gamma\hat{W} \quad (19)$$

where $F \in R^{m\times p}$, $\Gamma = \Gamma^T > 0$ and $\sigma > 0$ are design parameters.

From (1) and (18), the observer error can be given as

$$\begin{cases}\dot{\tilde{x}}(t)=A_0\tilde{x}+B[f(x(t),u(t))+h(x(t-\tau))-\hat{W}^T\Phi(\hat{x}(t),u(t))] \\ \tilde{y}(t)=C\tilde{x}(t)\end{cases} \quad (20)$$

Remark 3: Compared to the observer developed in Section III, the observer proposed in (18) does not include the robust term (5), then the matching condition $PB=C^TF^T$ (Assumption 1) is not needed in this case, which provides more design freedom. The σ -modification parameter in (19) is selected as a constant σ in this section.

We are now in the position to provide the following result:

Theorem 2: Consider the adaptive observer (18) with the NN adaptation law (19) for system (1), then for parameters $0\le 2lk\le\sigma$, $\lambda_{\mathrm{m}}(Q)\ge(3\|PB\|^2+\|FC\|^2)/k$, all signals in the observer remain semi-globally uniformly ultimately bounded (SGUUB).

Proof: We select a Lyapunov function candidate as

$$\bar{V}=\frac{1}{2}\tilde{x}^TP\tilde{x}+\frac{ke^{\varpi\tau_M}}{2(1-\bar{\tau})}\sum_{i=1}^{m}\int_{t-\tau_i}^{t}e^{-\varpi(t-\varsigma)}h_i^2(x(\varsigma))d\varsigma+\frac{1}{2}tr(\tilde{W}^T\Gamma^{-1}\tilde{W}) \quad (21)$$

where $\varpi>0,k>0$ are design parameters.

By using Young's inequality and (19) ~ (20), the derivative of $\bar{V}$ can be given as

$$\begin{aligned}\dot{\bar{V}}\le&-\frac{1}{2}\left(\lambda_m(Q)-\frac{1}{k}\|PB\|^2\right)\|\tilde{x}\|^2+\tilde{x}^TPB\left(Q(Z)-\hat{W}^T\Phi(\hat{x},u)\right)\\ &+tr(\tilde{W}^T\Gamma^{-1}\dot{\hat{W}})+\left[1-2\tanh^2\left(\frac{\|\tilde{x}^TPB\|^2}{\omega}\right)\right]\frac{ke^{\tau_M}h^T(x)h(x)}{2(1-\bar{\tau})}-\varpi V_3\end{aligned} \quad (22)$$

where $Q(Z)=f(x,u)+\dfrac{ke^{\tau_M}B^TP\tilde{x}}{(1-\bar{\tau})\|\tilde{x}^TPB\|^2}\tanh^2(\dfrac{\|\tilde{x}^TPB\|^2}{\omega})h^T(x)h(x)$

is an unknown function with $\omega>0$. Similar to (12), $Q(Z)$ can be approximated by a HONN on a compact set.

Since $\Phi^T\Phi\le l$, the following inequalities hold for $k>0$

$$\begin{aligned}&\tilde{x}^TPB(\tilde{W}^T\Phi+\tilde{\varepsilon})\le\frac{1}{k}\|PB\|^2\|\tilde{x}\|^2+\frac{kl}{2}\|\tilde{W}\|^2+\frac{k}{2}\varepsilon_N^2,\\ &-tr(\tilde{W}^T\Phi\tilde{y}^TF^T)\le\frac{1}{2k}\|FC\|^2\|\tilde{x}\|^2+\frac{kl}{2}\|\tilde{W}\|^2.\end{aligned} \quad (23)$$

Then the inequality (22) can be rewritten as

$$\begin{aligned}\dot{\bar{V}}\le&-\frac{1}{2}\left(\lambda_m(Q)-\frac{3}{k}\|PB\|^2-\frac{1}{k}\|FC\|^2\right)\|\tilde{x}\|^2\\ &-\frac{1}{2}(\sigma-2kl)\|\tilde{W}\|^2+\frac{1}{2}\sigma W_N^2+\frac{k}{2}\varepsilon_N^2-\varpi V_3\\ &+\frac{ke^{\varpi\tau_M}}{2(1-\bar{\tau})}\left[1-2\tanh^2\left(\frac{\|\tilde{x}^TPB\|^2}{\omega}\right)\right]h^T(x(t))h(x(t))\\ \le&-\eta_2\bar{V}+\beta_2+\left[1-2\tanh^2\left(\frac{\|\tilde{x}^TPB\|^2}{\omega}\right)\right]\frac{ke^{\varpi\tau_M}h^T(x(t))h(x(t))}{2(1-\bar{\tau})}\end{aligned} \quad (24)$$

with $\eta_2=\min\left\{\dfrac{\lambda_m(Q)-3\|PB\|^2/k-\|FC\|^2/k}{\lambda_M(P)},\dfrac{\sigma-2kl}{\lambda_M(\Gamma^{-1})},\varpi\right\}$, and $\beta_2=\dfrac{1}{2}\sigma W_N^2+\dfrac{k}{2}\varepsilon_N^2$, which are positive constant by setting the parameters σ, k and $\lambda_m(Q)$ large enough.

Similar to Section III, the last term in (24) is also bounded in a compact set. Then according to Lyapunov's Theorem, $\bar{V}$ is semi-globally uniformly ultimately bounded (SGUUB). Recalling (21), we know $\tilde{x}$ and $\tilde{W}$ are all bounded, which further guarantees the boundedness of $\hat{x}$, $\hat{W}$ and $\hat{y},\tilde{y}$. □

Remark 4: For the case $\|\tilde{x}^TPB\|^2\ge 0.8814\omega$, according to Lemma 2, the last term in (24) is negative, then the Lyapunov function (24) can be reduced as $\dot{\bar{V}}\le-\eta_2\bar{V}+\beta_2$ .This can improve the error convergence rate. Moreover, the observer error can be adjusted appropriately small by tuning the design parameters. Generally, the error can be minimized by small enough σ, ω, k and by large $\lambda_m(Q)$, Γ and ϖ.

V. Simulation Examples

Example 1: Consider the following nonlinear MIMO system which is similar to [6] but includes the delayed dynamics

$$\begin{cases}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\underbrace{\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}}_{A}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}+\underbrace{\begin{bmatrix}0&0\\1&0\\0&0\\0&1\end{bmatrix}}_{B}\left\{\underbrace{\begin{bmatrix}(1-x_1^2)x_2-x_1+x_3u\\(1-x_3^2)x_4-x_3+1/(1+x_2^2x_4^2)u\end{bmatrix}}_{f(x(t),u(t))}\right.\\ \qquad\left.+\underbrace{\begin{bmatrix}2\cos(x_1(t-\tau))\\2(x_1^2(t-\tau)+x_2^2(t-\tau))\sin(x_2(t-\tau))\end{bmatrix}}_{h(x(t-\tau))}\right\}\\ y(t)=\begin{bmatrix}y_1(t)\\y_2(t)\end{bmatrix}=\underbrace{\begin{bmatrix}1&1&0&0\\0&0&1&1\end{bmatrix}}_{C}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\\x_4(t)\end{bmatrix}\end{cases} \quad (25)$$

Under a bounded input $u(t)=\sin(0.3t)$ and a time-delay $\tau(t)=0.5(1+\sin(t))$ (i.e., $\tau_M=1,\bar{\tau}=0.5$), the system states $x(t)$ are bounded. In the first case, to fulfill the matching condition $PB=C^TF^T$, we can select

$$P=\begin{bmatrix}70&35&0&0\\35&35&0&0\\0&0&70&35\\0&0&35&35\end{bmatrix},\ K=\begin{bmatrix}1&0\\10&0\\0&1\\0&10\end{bmatrix},\ F=\begin{bmatrix}35&0\\0&35\end{bmatrix}.$$

The initial conditions are chosen as $x(0)=[1,1,1,1]^T$, $\hat{x}(0)=[3,-3,-3,3]^T$, $\hat{W}_1(0)=\hat{W}_2(0)=[0,\cdots,0]^T\in\mathbb{R}^8$, $\hat{\varepsilon}(0)=10$. The basis functions and the learning parameters are set as $\sigma(x)=2/(1+e^{-0.1x})-1$, $\Gamma=diag(200,200,\cdots,200)$, $\Upsilon=0.5$ and $\sigma_1(t)=0.01e^{-0.01t},\sigma_2(t)=10e^{-0.01t},\sigma_3(t)=e^{-0.01t}$. The condition $\delta(t)\le\bar{\delta}(t)$ in the robust compensation term (5) is also satisfied by choosing $\delta(t)=1e^{-0.01t}$. Then the observer (4) can be given as

$$\begin{cases}\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\\ \dot{\hat{x}}_3\\ \dot{\hat{x}}_4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \hat{x}_3\\ \hat{x}_4\end{bmatrix}+\begin{bmatrix}0&0\\1&0\\0&0\\0&1\end{bmatrix}\left\{\begin{bmatrix}\hat{W}_1^T\Phi(\hat{x}(t),u(t),\tilde{y}(t))\\ \hat{W}_2^T\Phi(\hat{x}(t),u(t),\tilde{y}(t))\end{bmatrix}\right.\\ \qquad\left.+\dfrac{F\tilde{y}}{2\delta(t)}+\dfrac{\hat{\varepsilon}^2F\tilde{y}}{\hat{\varepsilon}\,|\,F\tilde{y}\,\|+\sigma_2(t)}\right\}+\begin{bmatrix}1&0\\10&0\\0&1\\0&10\end{bmatrix}\begin{bmatrix}y_1(t)-\hat{y}_1(t)\\ y_2(t)-\hat{y}_2(t)\end{bmatrix}\\ \begin{bmatrix}\hat{y}_1\\ \hat{y}_2\end{bmatrix}=\begin{bmatrix}1&1&0&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \hat{x}_3\\ \hat{x}_4\end{bmatrix}\end{cases}\qquad(26)$$

The state responses are provided in Fig. 1, the dotted line is the state profile of observer and the solid line is the system state. The observer errors are depicted in Fig. 2. It is shown that the errors converge to the origin after a small transient and a satisfactory observer performance can be achieved.

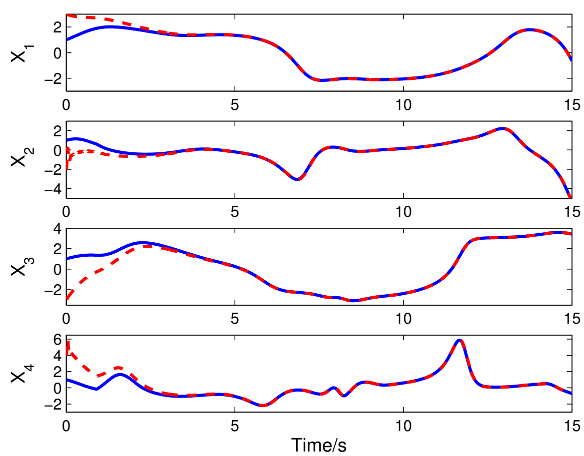

Fig. 1. State profile with robust term.

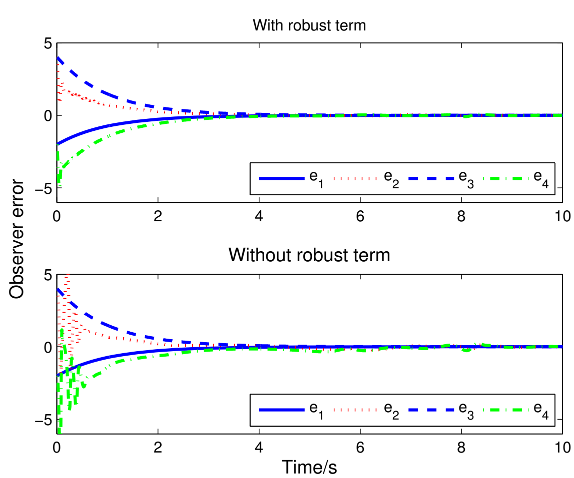

Fig. 2. Observer errors with (without robust term).

The adaptive observer (18) without the robust term (5) is also tested. In this case, we choose $F=\begin{bmatrix}40&1\\-1&40\end{bmatrix}$ and $\sigma=0.001$. Other simulation parameters are the same as previous ones. It is verified that the matching condition $PB=C^TF^T$ cannot be fulfilled. The observer errors are also depicted in Fig. 2, which shows that the observer errors also converge to a small region around the origin though a more sluggish transient is provided.

Example 2: For comparison, the nonlinear time-delay system used in [11] is utilized without the output nonlinearity

$$\begin{cases}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\underbrace{\begin{bmatrix}-1.8&0.2&-0.5\\-0.3&-2.6&0.9\\-0.3&0.7&-2.4\end{bmatrix}}_{A}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}+\underbrace{\begin{bmatrix}0.5\cos(x_2(t)+x_3(t))\\0\\-0.6\cos(x_1(t)-x_2(t))\end{bmatrix}}_{f(x(t))}\\ \qquad+\underbrace{\begin{bmatrix}0.04&-0.01&-0.01\\0.01&-0.03&0.02\\0.01&-0.01&0.05\end{bmatrix}\begin{bmatrix}x_1(t-\tau)\\ x_2(t-\tau)\\ x_3(t-\tau)\end{bmatrix}}_{h(x(t-\tau))}\\ y(t)=\underbrace{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}}_{C}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}\end{cases}$$

(27)

where the time delay is $\tau(t)=|0.4\sin(t)|$ with $\tau_M=\overline{\tau}=0.5$. The design matrix P and the observer gain matrix K are specified as [11]. For the first case with robust term, it is known that the matching condition $PB=C^TF^T$ can be fulfilled by selecting $B=C=I^{3\times3}$ and $P=F^T$. The HONN learning rate is $\Gamma=diag(20,20,\cdots,20)$ and other simulation parameters are the same as in Example 1. For the second case without robust compensation, the matching condition is not necessarily fulfilled, then we set $F=I^{3\times3}$ and $\sigma=0.001$ to validate the generality of our method. For comparison, the observer error performance of the scheme proposed in [11] is also provided.

The corresponding observer errors of different observers are depicted in Fig. 3 with initial conditions $x(0)=[0,0,0]^T$, $\hat{x}(0)=[-2,3,2]^T$, in which the error convergence can be verified. As can be seen from Fig. 3, both of the proposed observers have a better transient performance with shorter duration than [11], and achieve an equivalent steady state performance. The observer with robust compensation has a slightly better convergence performance. However, it should be mentioned that the precise system model including the time-delay $\tau(t)$, the nonlinearities $f(x(t),u(t))$ and the delayed dynamics $h(x(t-\tau))$ of system (27) are assumed to be known in [11], which are not required in this paper.

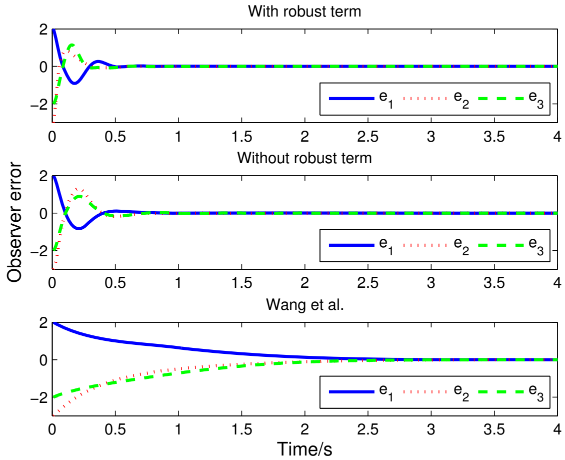

Fig. 3. Comparative observer errors.

VI. CONCLUSION

This paper proposes two adaptive neural observers for a class of nonlinear unknown time-delay systems. The unknown time-varying delays are not involved in the observers with the help of novel Lyapunov-Krasovskii function designs, such that the presented schemes can be mentioned as memoryless observers. The precise system model or other restrictive assumptions of nonlinear system functions which are usually required in the observer design are not utilized. The observer error of the first observer with an adaptive robust compensation term is guaranteed to be asymptotically stable, and the error of the second observer without using the matching condition is proved to be semi-globally uniformly ultimately bounded. Comparative simulations are conducted to illustrate the reliability of the proposed methods.

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